Notes from the HW due 2009-09-08

§ 2.4 #1

(a) Find the largest open interval, centered at the origin on the x-axis, such that for each x in the interval the value of the function $f(x) = x + 2$ is within 0.1 units of the number $f(0) = 2$

Ans

Here we are talking about a y-axis interval (0.1 within $y = 2$). So, we are considering values of y that fall in the interval (1.9, 2.1).

What is the largest x-interval that will send values of x to values of $f(x)$ in the interval (1.9, 2.1)? Well … which x is "sent to" 1.9?

$$1.9 = x + 2 \Rightarrow x = -.1$$

Which x is "sent to" 2.1?

$$2.1 = x + 2 \Rightarrow x = +.1$$

So … all the x values in our x interval of (-0.1, +0.1) are "sent, by f," into (1.9, 2.1).

In terms of section 2.4 and the material we did in class, $\varepsilon = 0.1$ (a 'y' distance) and $\delta = 0.1$ (an 'x' distance).

(b) Find the largest open interval, centered at $x = 3$, such that for each x in the interval the value of the function $f(x) = 4x - 5$ is within 0.01 units of the number $f(3) = 7$.

Ans

7 – 0.01 = 6.99; 7 + 0.01 = 7.01; So we want $f(x)$ to fall within (6.99, 7.01).

$$6.99 = 4x_1 - 5 \Rightarrow x_1 = 2.9975$$
$$7.01 = 4x_2 - 5 \Rightarrow x_2 = 3.0025$$

So … the associated x-interval to our y-interval of (6.99, 7.01) (corresponding to an ε of 0.01) is (2.9975, 3.0025) (corresponding to a δ of .0025).

Note that the book's notation for this is that this is 'the interval of all the x's such that $|x - 3| < 0.0025$ - i.e. the x's whose <u>distance from $x = 3$</u> is less than 0.0025. If you solve this inequality, you'll see that the intervals are the same.

(c) Find the largest open interval, centered at $x = 4$, such that for each x in the interval the value of the function $f(x) = x^2$ is within 0.001 units of the number $f(4) = 16$.

Ans

Quick version ….

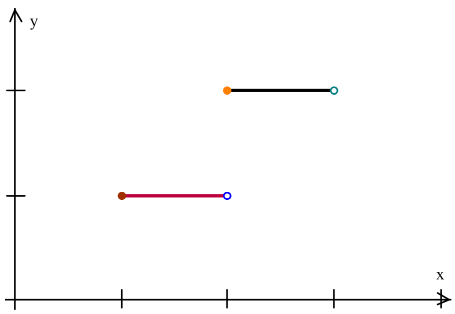
Solve $x^2 = 15.999$ for x_1 ($x_1 = 3.99987$)

Solve $x^2 = 16.001$ for x_2 ($x_2 = 4.00012$)

The largest interval (of x values) is (x_1, x_2) – in other words (3.99987, 4.00012).

The work for the other problems in 2.4 is similar ….

§ 2.5 #1



On which of the following intervals (if any) is the graphed function continuous?

(a) $[1,3]$ - the square brackets indicate an interval that includes the endpoints. So – the 'inequality notation' would be $1 \le x \le 3$. You can see the function is not continuous at $x = 2$, which *is* included in this interval, so the answer would be **not continuous**. If you wish to think about this in terms of our 3-condition definition of continuity (and you should ….), continuity fails at $x = 2$ because the **2-sided limit** as x approaches 2 **does not exist**.

(b) $(1,3)$ - this is the interval that does not include either endpoint. (Interval notation: $1 < x < 3$). Continuity fails for the same reason as above.

(c) $[1,2]$ - Continuity **STILL fails** here, at $x = 2$. The point $x = 2$ is included in our interval in question, and you would have to 'jump up' from the line segment to reach it. So again, the 2-sided limit fails to exist.

(d) $(1,2)$ - The function **is continuous** on this interval. The entire interval is contained in the first line segment, and so

* The function is defined at each point
* The 2-sided limit exists at each point
* And those two numbers are the same.

(e) and (f) I'll let you try!!

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§ 2.4 #9

Given $f(x) = 2x;\ \lim_{x\to 4} 2x = 8;\ \varepsilon = 0.1$ find a number δ such that $|f(x) - L| < \varepsilon \ if\ 0 < |x-4| < \delta$.

Ans

So the epsilon we are given (0.1) defines an interval on the y-axis that we are aiming for. That interval is (7.9, 8.1). What delta **guarantees** that $f(x)$ is within that interval for every x that is within delta of 4?

Solve for the x endpoints (really, I'm interested in the minimum and maximum values of y in that interval – on a line they are going to be at the endpoints).

$7.9 = 2x_1 \Rightarrow x_1 = 3.95;\quad 8.1 = 2x_2 \Rightarrow x_2 = 4.05$

4 – 3.95 = 0.05 and 4.05 – 4 = 0.05. I want a δ that is less that or equal to either of those (identical) quantities, so …

Choose $0 < \delta < 0.05$.